

1 **A Supplementary material for LHCb-PAPER-2024-**
2 **028**

3 This appendix contains supplementary material that will be posted on the public CDS
4 record but will not appear in the paper.

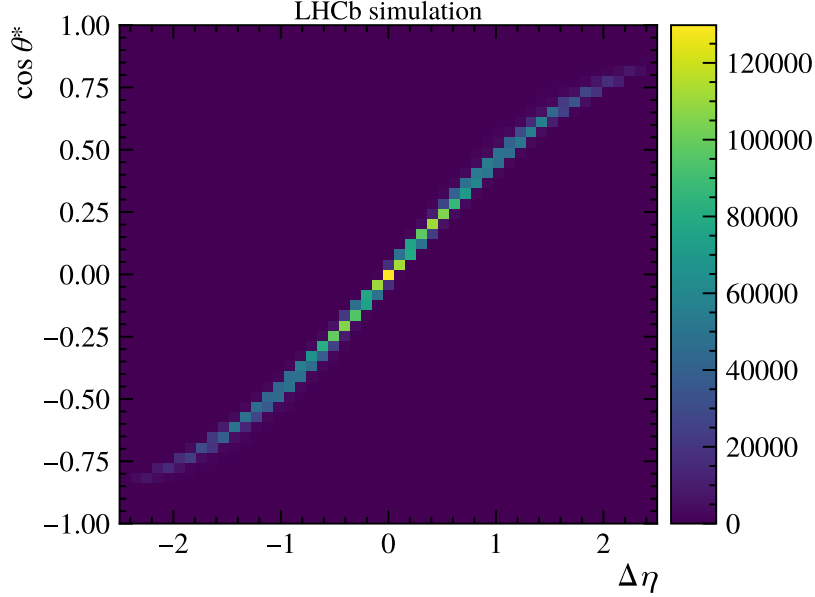


Figure 1: Distribution of $\cos \theta^*$ versus $\Delta\eta$ in simulated $Z \rightarrow \mu^+ \mu^-$ decays.

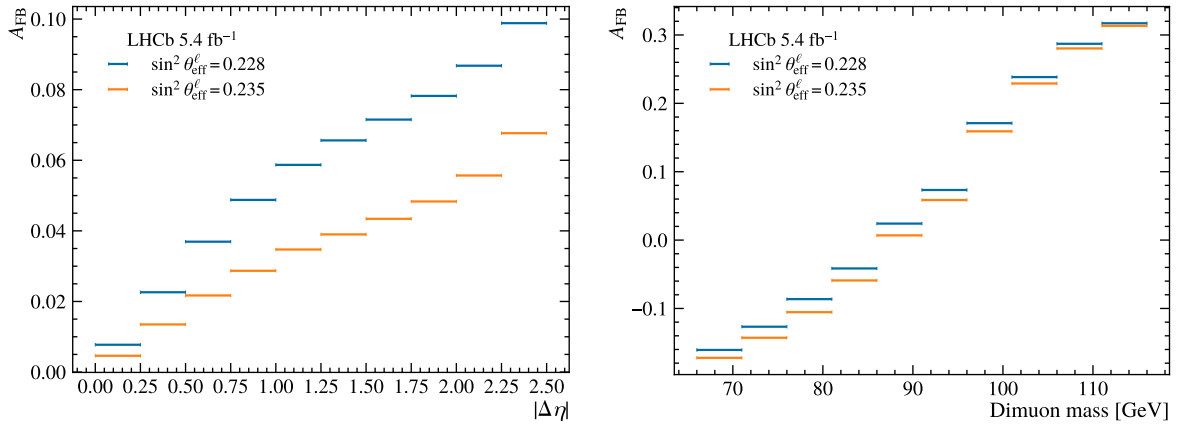


Figure 2: Predictions of A_{FB} in intervals of (left) $|\Delta\eta|$ and (right) mass for two different $\sin^2 \theta_{\text{eff}}^\ell$ values.

5 Figure 1 shows that $\Delta\eta$ is closely related to $\cos \theta^*$, following the approximation $\cos \theta^* \sim$
6 $\tanh(\Delta\eta/2)$ is, thereby demonstrating why the sensitivity to $\sin^2 \theta_{\text{eff}}^\ell$ is improved by finding
7 A_{FB} as a function of $|\Delta\eta|$. Figure 2 shows how predictions of A_{FB} change as a function of
8 $|\Delta\eta|$ or the dimuon invariant mass when the value of $\sin^2 \theta_{\text{eff}}^\ell$ is varied, thereby showing
9 the power of measuring A_{FB} as a function of $|\Delta\eta|$. Figure 3 shows the absolute variation
10 in A_{FB} corresponding to the NNPDF 3.1 parton distribution function uncertainties and
11 to a variation in $\sin^2 \theta_{\text{eff}}^\ell$. Figure 4 shows the χ^2 parabola associated with the fit to A_{FB}
12 to determine the weak mixing angle. Figure 5 shows the mass and $|\Delta\eta|$ distributions
13 of the selected signal candidates, compared to the simulation including the signal and
14 backgrounds.

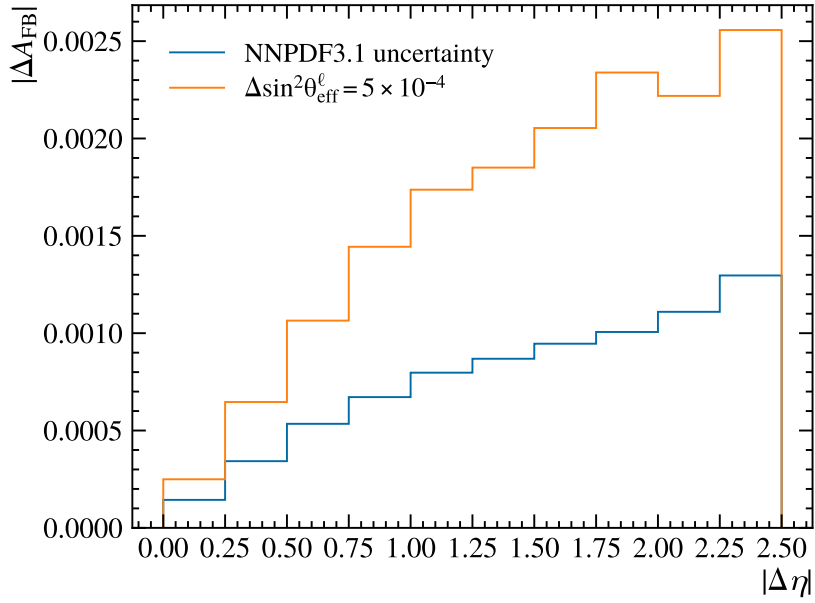


Figure 3: Absolute variation in A_{FB} , in intervals of $|\Delta\eta|$, corresponding to the uncertainty on the NNPDF3.1 parton distribution functions and to a variation in $\sin^2\theta_{\text{eff}}^\ell$.

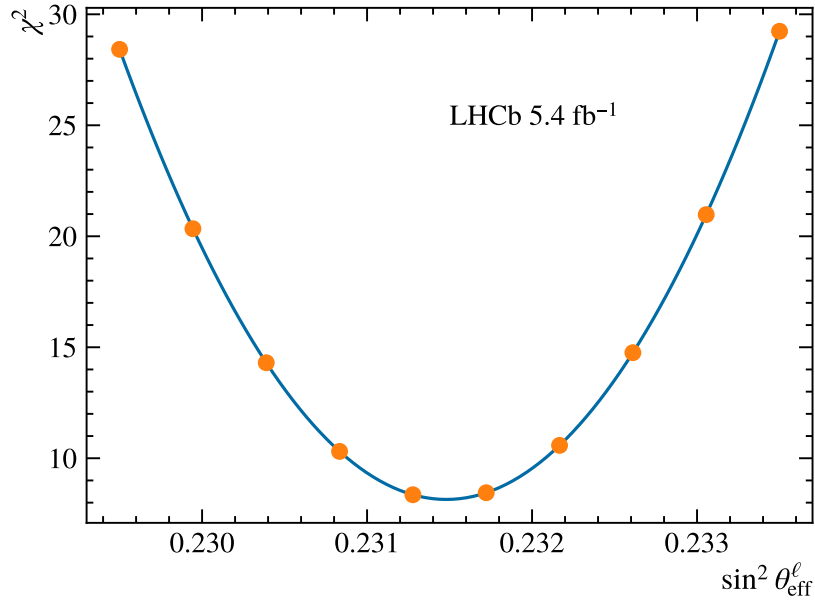


Figure 4: Graph of the χ^2 values comparing the measured A_{FB} with the interpolated templates, using the POWHEG-ewnl0 predictions with the NNPDF3.1 PDF parameterisation. A parabolic fit is overlaid. This corresponds to using two base templates, with linear interpolation of the A_{FB} predictions to intermediate values.

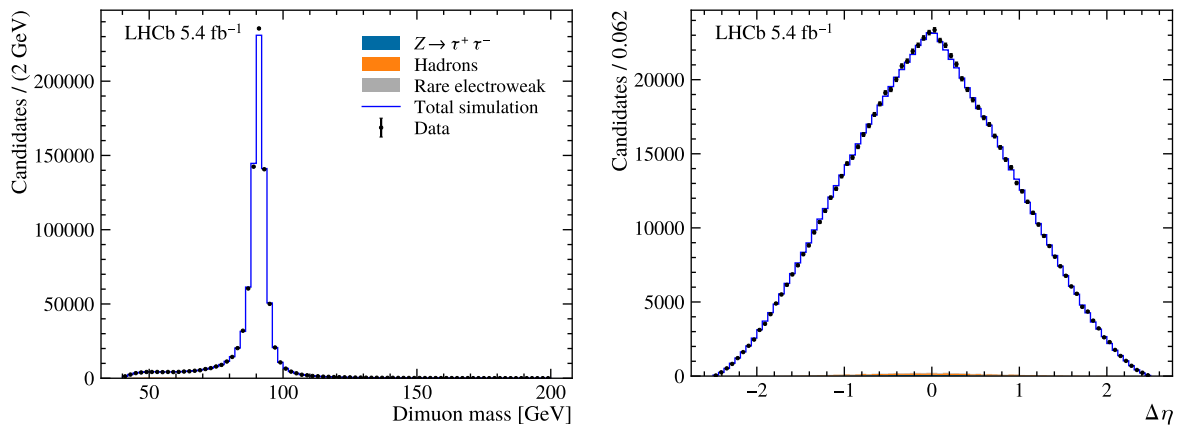


Figure 5: Distributions of (left) the dimuon invariant mass and (right) $\Delta\eta$ for the selected signal candidates, compared to the simulation including the signal and backgrounds.